Russian Federal Nuclear Center –
Zababakhin Institute of Applied Physics

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“ROSATOM” STATE CORPORATION

# An equivalent inverse source method of evaluating the weight functions for parallel-plate time-domain diffuse optical tomography

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# Contents

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# Lyubimov's perturbation model

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## ➢ The fundamental equation

$$g(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d)=\int_V [W_{\mu_a}(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r})\,\delta\mu_a(\mathbf{r})+W_D(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r})\,\delta D(\mathbf{r})]d^3r,$$

$$g(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d)=\frac{\Gamma_0(\mathbf{r}_s,t_s,\mathbf{r}_d,t)\big|_{t=t_d}-\Gamma(\mathbf{r}_s,t_s,\mathbf{r}_d,t)\big|_{t=t_d}}{\Gamma_0(\mathbf{r}_s,t_s,\mathbf{r}_d,t)\big|_{t=t_d}}$$

**is the time-resolved optical projection,**

$\Gamma(\mathbf{r}_s,t_s,\mathbf{r}_d,t)$ **is the temporal point spread function**

## ➢ The weight functions

$$W_{\mu_a}(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r})=c\int_{t_s}^{t_d}\frac{G(\mathbf{r}-\mathbf{r}_s,t-t_s)\,\partial G(\mathbf{r}_d-\mathbf{r},t_d-t)/\partial\eta}{\partial G(\mathbf{r}_d-\mathbf{r}_s,t_d-t_s)/\partial\eta}dt$$

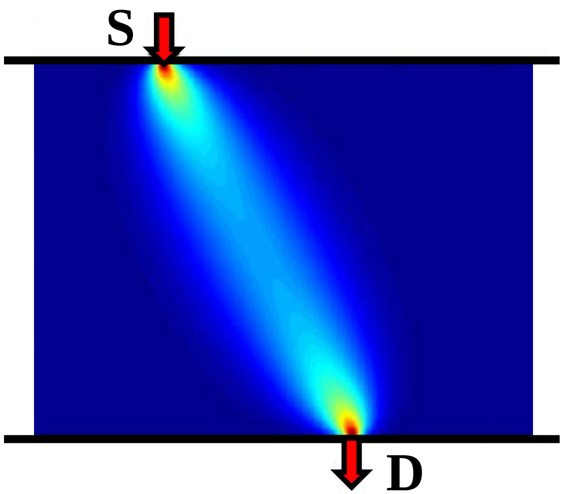
$$W_D(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r})=-\frac{1}{D}\int_{t_s}^{t_d}\frac{G(\mathbf{r}-\mathbf{r}_s,t-t_s)\partial G(\mathbf{r}_d-\mathbf{r},t_d-t)/\partial\eta}{\partial G(\mathbf{r}_d-\mathbf{r}_s,t_d-t_s)/\partial\eta}\left[c\mu_a+\frac{\partial}{\partial t}\ln G(\mathbf{r}-\mathbf{r}_s,t-t_s)\right]dt$$

V.V. Lyubimov, Opt. Spectrosc. 80: 616 (1996)

A.B. Konovalov et al., Proc. SPIE 8088: 80880T (2011)

# Two-step approach for weight function evaluation

## ❑ Our approach is based on two assumptions

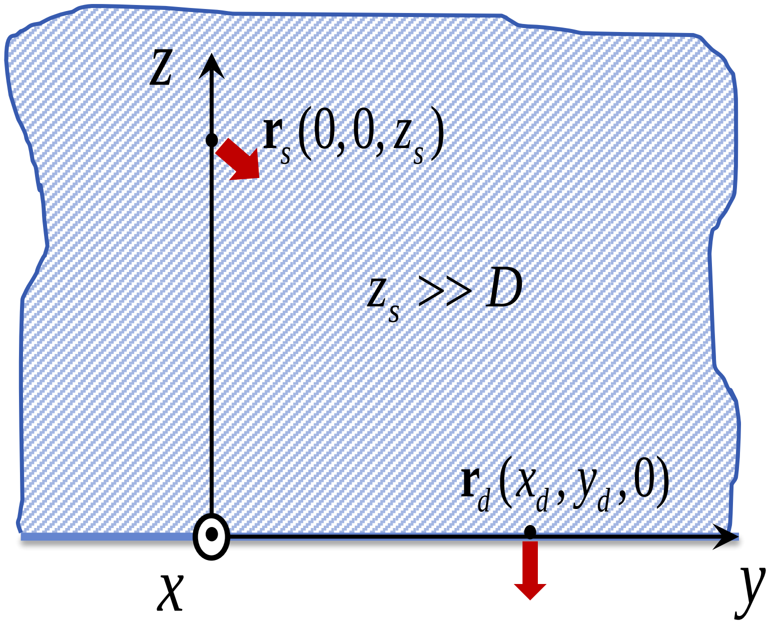



- The effect of one boundary on photon migration near the other boundary is negligibly small.
- The distributions of photons near the source S on one boundary and near the receiver D on the other boundary are similar in nature.

## ❑ Our approach involves two steps

- Derive the analytical expressions for the case of semi-infinite scattering medium
- Apply them to obtain the weight functions for the flat layer using the equivalent inverse source method

# 3D case: half-space $\{(x, y, z) : z \geq 0\}$




$$W_{\mu_a}(\ldots) = c\int_{t_s}^{t_d} \frac{\boldsymbol{G}(\ldots)\partial \boldsymbol{G}(\ldots)/\partial\eta}{\partial \boldsymbol{G}(\ldots)/\partial\eta} dt$$

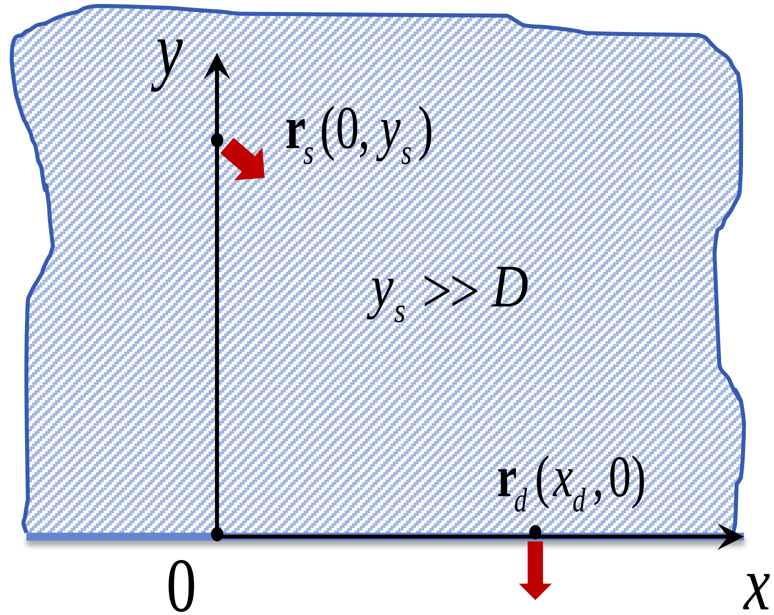
$$W_D(\ldots) = -\frac{1}{D}\int_{t_s}^{t_d} \frac{\boldsymbol{G}(\ldots)\partial \boldsymbol{G}(\ldots)/\partial\eta}{\partial \boldsymbol{G}(\ldots)/\partial\eta} \times \left[ c\mu_a + \frac{\partial}{\partial t}\ln \boldsymbol{G}(\ldots) \right] dt$$

## 3D Green's function

$$\boldsymbol{G}^{3D}(\mathbf{r}-\mathbf{r}', t-t') = \left[4\pi Dc(t-t')\right]^{-3/2} \exp\left[-\mu_a c(t-t')\right]$$

$$\times\left\{ \exp\left[-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4Dc(t-t')}\right] - \exp\left[-\frac{(x-x')^2+(y-y')^2+(z+z')^2}{4Dc(t-t')}\right] \right\}$$

# 2D case: half-plane $\{(x,y): y \geq 0\}$




$$W_{\mu_a}(\ldots) = c\int_{t_s}^{t_d} \frac{\boldsymbol{G}(\ldots)\partial \boldsymbol{G}(\ldots)/\partial\eta}{\partial \boldsymbol{G}(\ldots)/\partial\eta} dt$$

$$W_D(\ldots) = -\frac{1}{D}\int_{t_s}^{t_d} \frac{\boldsymbol{G}(\ldots)\partial \boldsymbol{G}(\ldots)/\partial\eta}{\partial \boldsymbol{G}(\ldots)/\partial\eta} \times \left[c\mu_a + \frac{\partial}{\partial t}\ln \boldsymbol{G}(\ldots)\right] dt$$

## 2D Green's function

$$\boldsymbol{G}^{\mathrm{2D}}(\mathbf{r}-\mathbf{r}', t-t') = \left[4\pi Dc(t-t')\right]^{-1} \exp\left[-\mu_a c(t-t')\right] \times \left\{ \exp\left[-\frac{(x-x')^2+(y-y')^2}{4Dc(t-t')}\right] - \exp\left[-\frac{(x-x')^2+(y+y')^2}{4Dc(t-t')}\right] \right\}$$

# 3D case: half-space $\{(x,y,z): z \ge 0\}$

$$W_{\mu_a}(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r}) = W_{\mu_a}^{-} - W_{\mu_a}^{+}, \quad W_D(\mathbf{r}_s,t_s,\mathbf{r}_d,t_d,\mathbf{r}) = W_D^{+} - W_D^{-},$$ **where**

$$W_{\mu_a}^{\pm} = \frac{z}{\pi D z_s} \exp\left[\frac{x_d^2 + y_d^2 + z_s^2 - (\gamma + \beta_\pm)^2}{4Dct_d}\right]\left[\frac{1}{4\beta_\pm}\left(1 + \frac{\beta_\pm}{\gamma}\right)^2 + \frac{Dct_d}{2\gamma^3}\right],$$

$$W_D^{\pm} = \frac{z}{\pi D z_s} \exp\left[\frac{x_d^2 + y_d^2 + z_s^2 - (\gamma^2 + \beta_\pm^2)^2}{4Dct_d}\right]\left[\frac{\gamma^2}{16D^2c^2t_d^2\beta_\pm}\left(1 + \frac{\beta_\pm}{\gamma}\right)^4 + \frac{1}{8Dct_d\beta_\pm}\left(\frac{\beta_\pm^3}{\gamma^3} - 3\frac{\beta_\pm^2}{\gamma^2} - 9\frac{\beta_\pm}{\gamma} - 5\right) - \frac{3}{4\gamma^3}\right],$$

$$\gamma = \sqrt{(x - x_d)^2 + (y - y_d)^2 + z^2}, \quad \beta_\pm = \sqrt{x^2 + y^2 + (z \pm z_s)^2}$$

A.B. Konovalov, V.V. Vlasov, Quantum Electron. 44: 719 (2014)

A.B. Konovalov, V.V. Vlasov, Proc. SPIE 9917: 99170S (2016)

# 2D case: half-plane $\{(x,y): y \geq 0\}$

$$W_{\mu_a}(\mathbf{r}_s, t_s, \mathbf{r}_d, t_d, \mathbf{r}) = W_{\mu_a}^{-} - W_{\mu_a}^{+}, \quad W_D(\mathbf{r}_s, t_s, \mathbf{r}_d, t_d, \mathbf{r}) = W_D^{+} - W_D^{-},$$ **where**

$$W_{\mu_a}^{\pm} = \frac{y}{2y_s}\sqrt{\frac{ct_d}{\pi D}} \exp\left[\frac{x_d^2 + y_s^2 - (\gamma + \beta_\pm)^2}{4Dct_d}\right]\left(\frac{1}{\sqrt{\gamma\beta_\pm}} + \frac{\sqrt{\beta_\pm}}{\gamma^{3/2}}\right),$$

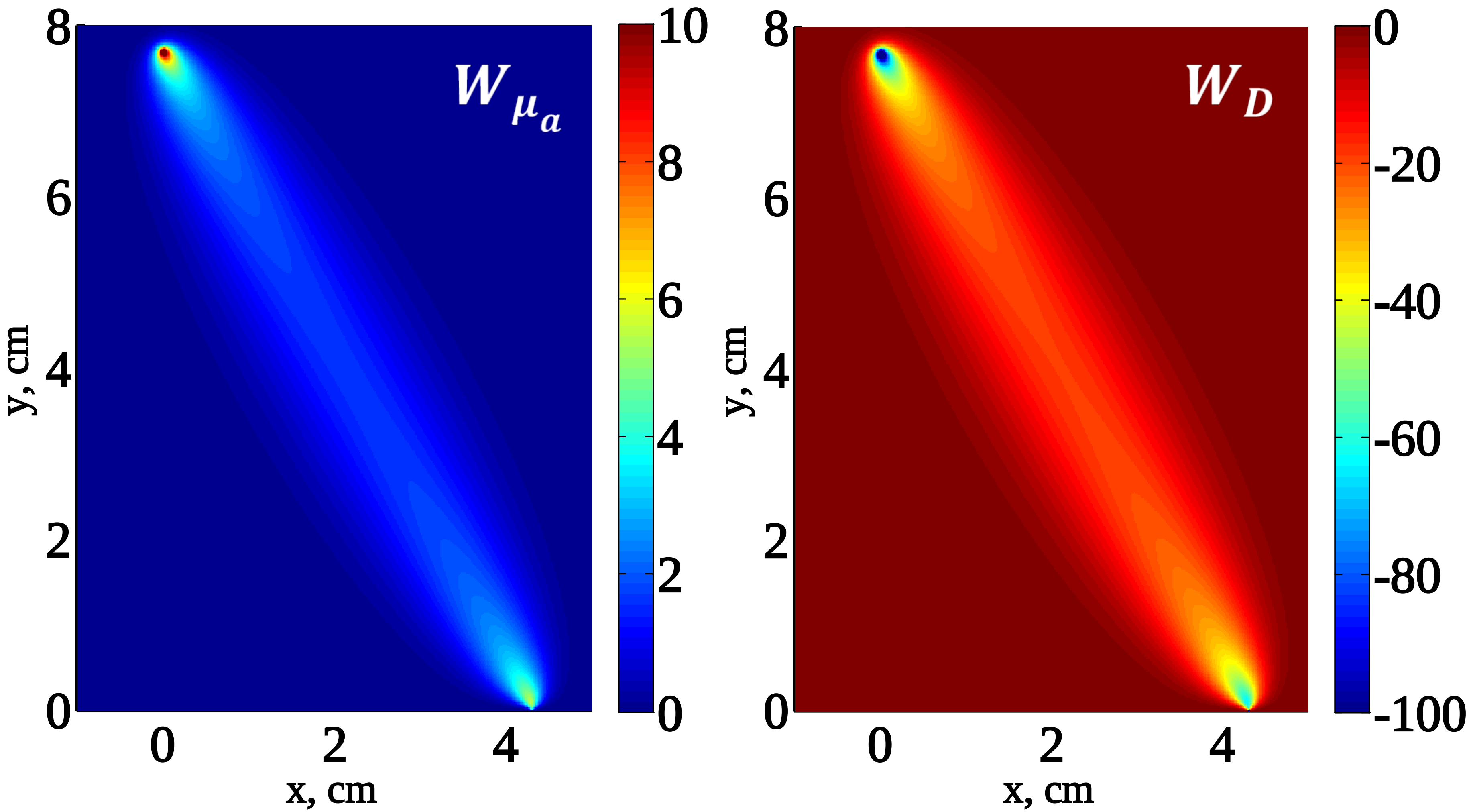
$$W_D^{\pm} = \frac{y}{D(4\pi Dct_d)^{3/2} y_s} \exp\left[\frac{x_d^2 + y_s^2 - (\gamma + \beta_\pm)^2}{4Dct_d}\right]$$
$$\times\left[\frac{1}{\sqrt{\gamma\beta_\pm}}\left(\gamma^2 + 3\beta_\pm^2 - 8Dct_d\right) + \frac{\sqrt{\beta_\pm}}{\gamma^{3/2}}\left(3\gamma^2 + \beta_\pm^2 - 4Dct_d\right)\right],$$

$$\gamma = \sqrt{(x - x_d)^2 + y^2}, \quad \beta_\pm = \sqrt{x^2 + (y \pm y_s)^2}$$


A.B. Konovalov, V.V. Vlasov, Quantum Electron. 44: 719 (2014)

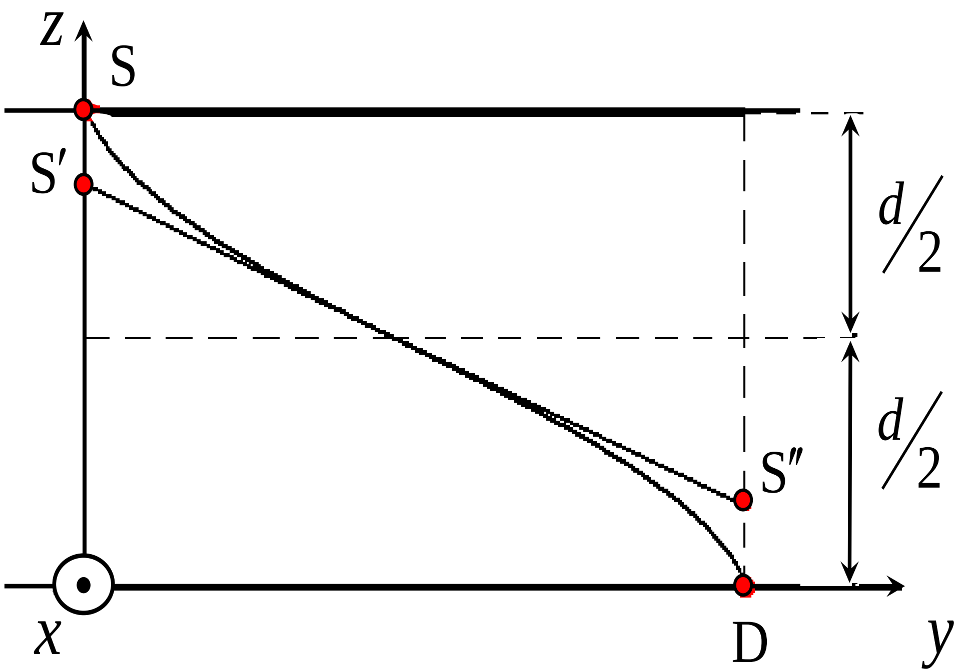
A.B. Konovalov, V.V. Vlasov, Proc. SPIE 9917: 99170S (2016)

# Examples of the weight functions



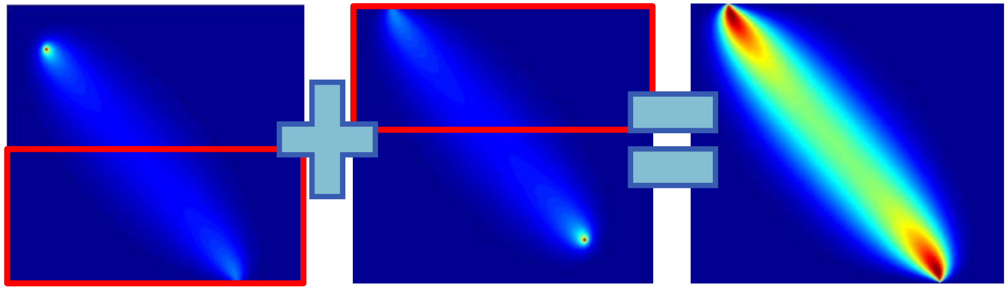

$D = 0.0332\ cm \qquad c = 0.0214\ cm/ps \qquad t_d = 1000\ ps$

# Method of the equivalent inverse source




- Find the position of S' inside the flat layer and calculate the weight function for one half of the layer.
- Find the position of S" and calculate the weight function for the other half of the layer.

- Perform superposition of the useful halves to obtain the weight function for the full layer.



# Finding the positions of the equivalent sources

**The position of the equivalent source S´ and the position of the equivalent inverse source S˝ are found from relations for the photon average trajectories.**

- **The coordinate** $z_{S'}$ **can be found by solving the equation**

$$Z\,|_{t=t_d/2} = d/2,$$ **where**

$$Z(t) = \left[\frac{2Dct}{z_{S'}} + \frac{z_{S'}(t_d - t)}{t_d}\right]\mathrm{erf}\left[\frac{z_{S'}^2(t_d - t)}{4Dctt_d}\right]^{1/2} + \left[\frac{4Dct(t_d - t)}{\pi t_d}\right]^{1/2}\exp\left[-\frac{z_{S'}^2(t_d - t)}{4Dctt_d}\right],$$

$\mathrm{erf}(\xi)$ **is the error function**

- **For the coordinate** $z_{S''}$ **we will have**

$$z_{S''} = d - z_{S'}$$

A.B. Konovalov et al., Quantum Electron. 36: 1048 (2006)

A.B. Konovalov et al., Optik 124: 6000 (2013)

# The weight function for a 2D flat layer

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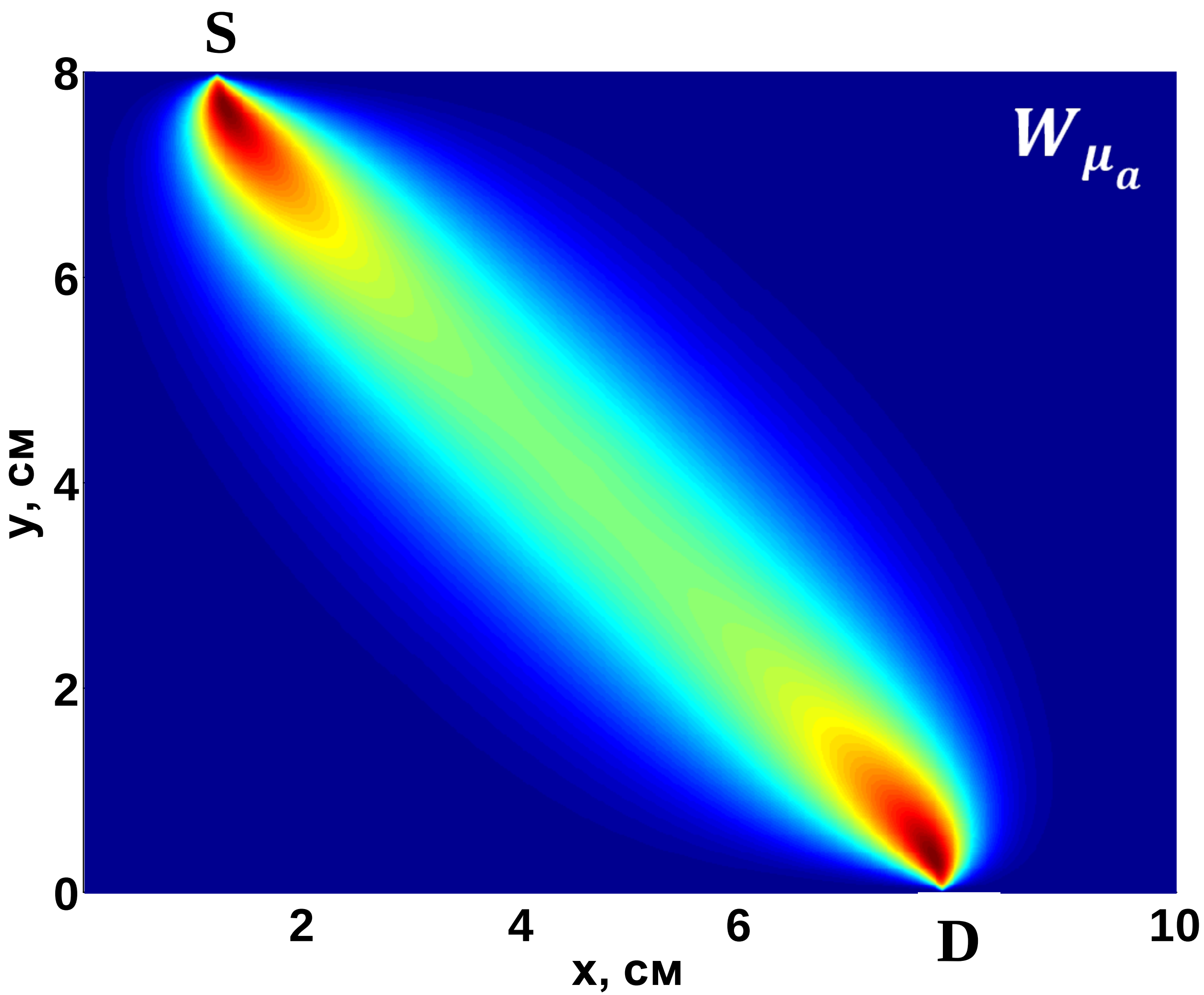


$$D = 0.0332\ cm$$

$$c = 0.0214\ cm/ps$$

$$t_d = 1000\ ps$$

$$\mathbf{r}_s(1.2, 8.0\ cm)$$

$$\mathbf{r}_d(7.8\ cm, 0)$$

# The reconstruction problem statement

**The reconstruction problem is reduced to the solution of the following system of linear algebraic equations.**

$$\begin{pmatrix} \mathbf{W}_{\mu_a} & \mathbf{W}_D \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta\mu}_a \\ \boldsymbol{\delta}\mathbf{D} \end{pmatrix} = \mathbf{g},$$ **where**

$\begin{pmatrix} \mathbf{W}_{\mu_a} & \mathbf{W}_D \end{pmatrix}$ **is the weight matrix obtained by sampling and combining all weight functions calculated for different positions of sources and receivers**

$\begin{pmatrix} \boldsymbol{\delta\mu}_a \\ \boldsymbol{\delta}\mathbf{D} \end{pmatrix}$ **is the vector of unknown local disturbances of the absorption and diffusion coefficients**

$\mathbf{g}$ **is the vector of time-resolved optical projections**

# Our reconstruction algorithm

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**To solve SLAE we use an algorithm that combines the algebraic reconstruction technique (ART) with regularization by total variation (TV) norm minimization.**

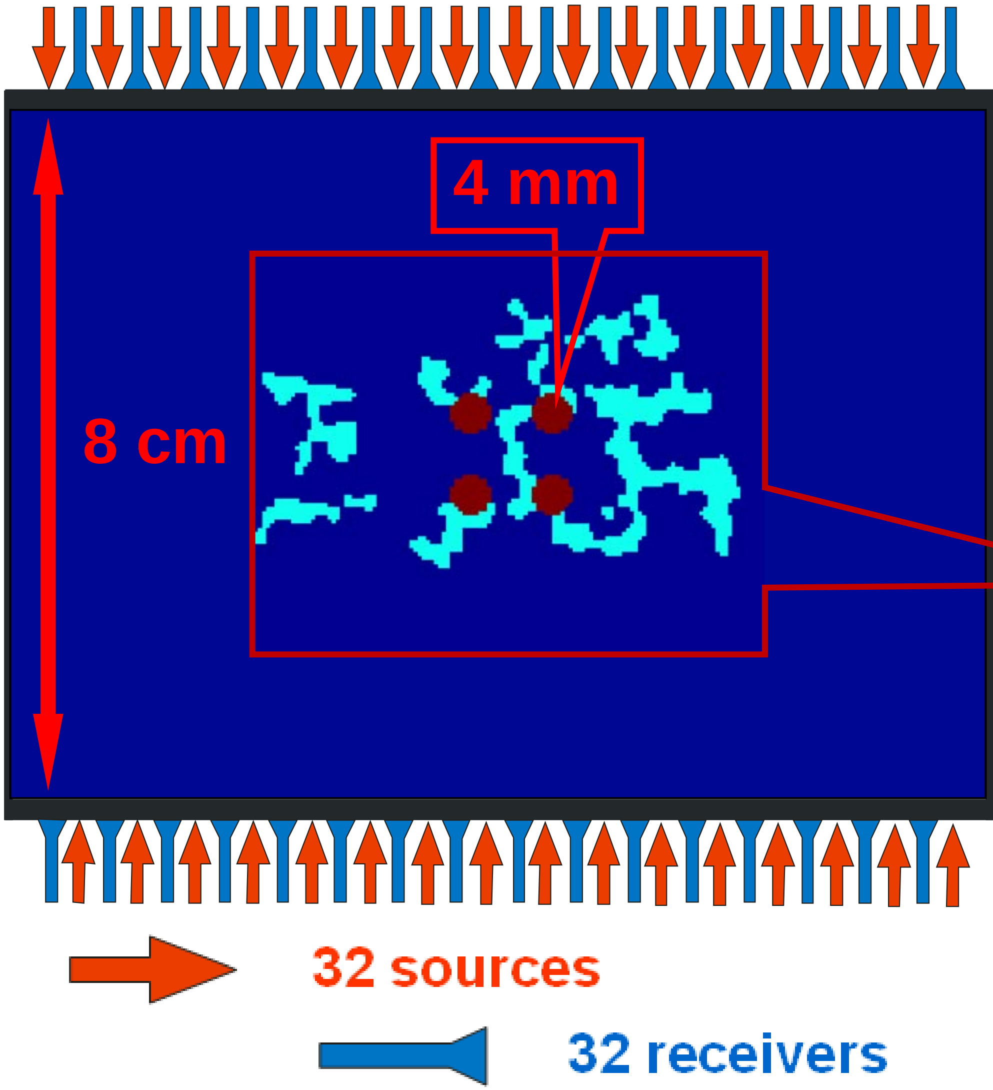
## ***Outline of the algorithm***

- ***Step 1***: **Do a number of ART iterations**
- ***Step 2***: **Process the result through TV-norm minimization**
- ***Step 3***: **Check the stop criterion. Go to *Step 1* if not satisfied**
- ***Step 4***: **Stop**

**The researcher interactively chooses when ART iterations should stop and TV-processing start. For the stop criterion, we use the iteration convergence rate.**

A.B. Konovalov, V.V. Vlasov, Proc. SPIE 9917: 99170S (2016)

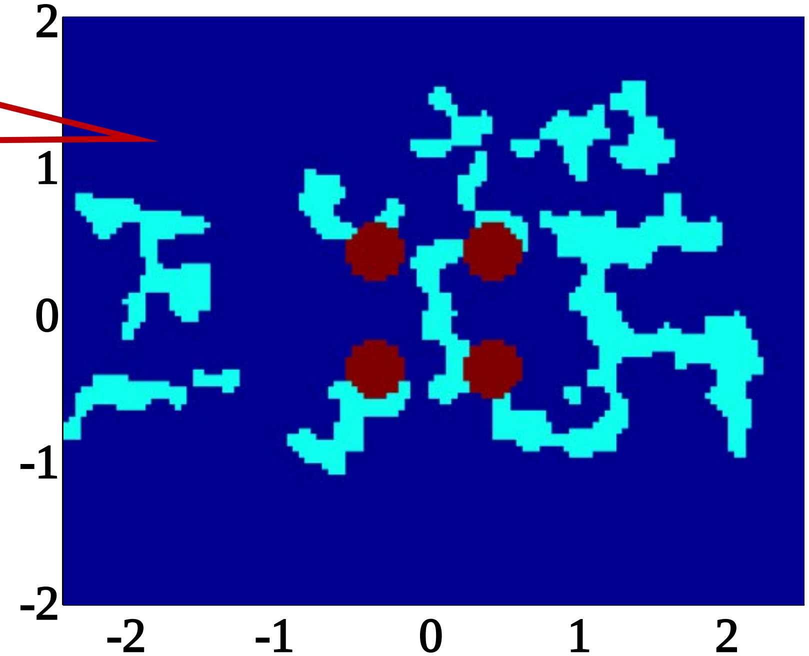
# The numerical phantom for study




$\mu_a = 0.05\ cm^{-1}$, $D = 0.0332\ cm$

$\mu_a = 0.075\ cm^{-1}$, $D = 0.0255\ cm$

$\mu_a = 0.06\ cm^{-1}$, $D = 0.0302\ cm$

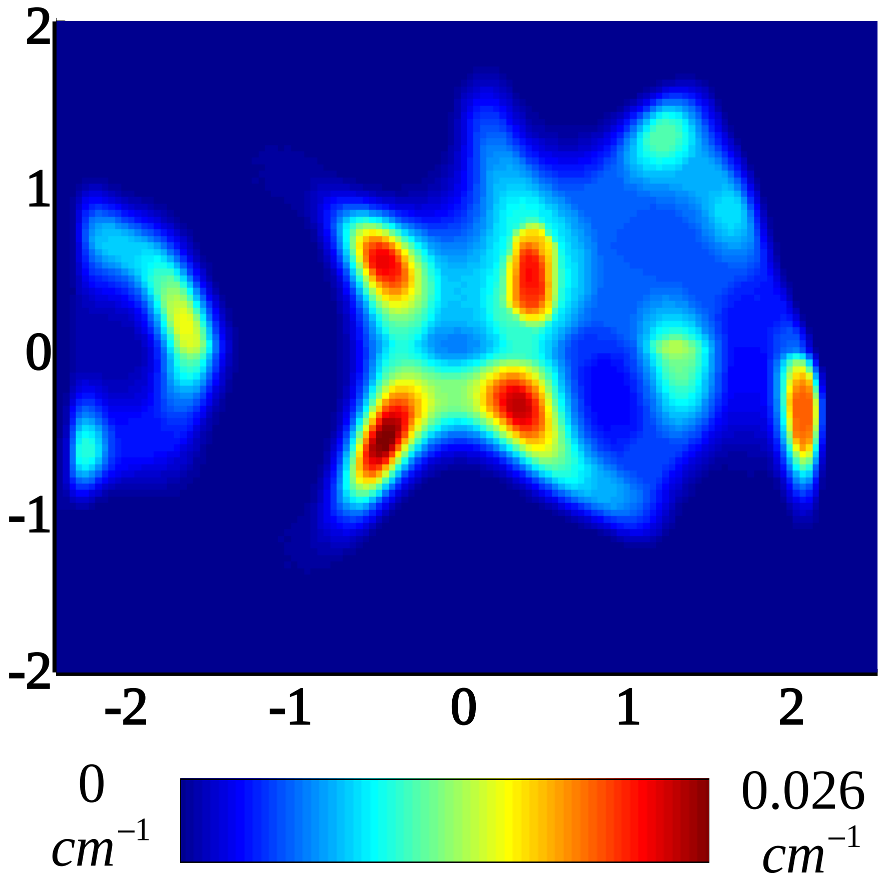


# The optical parameters we chose

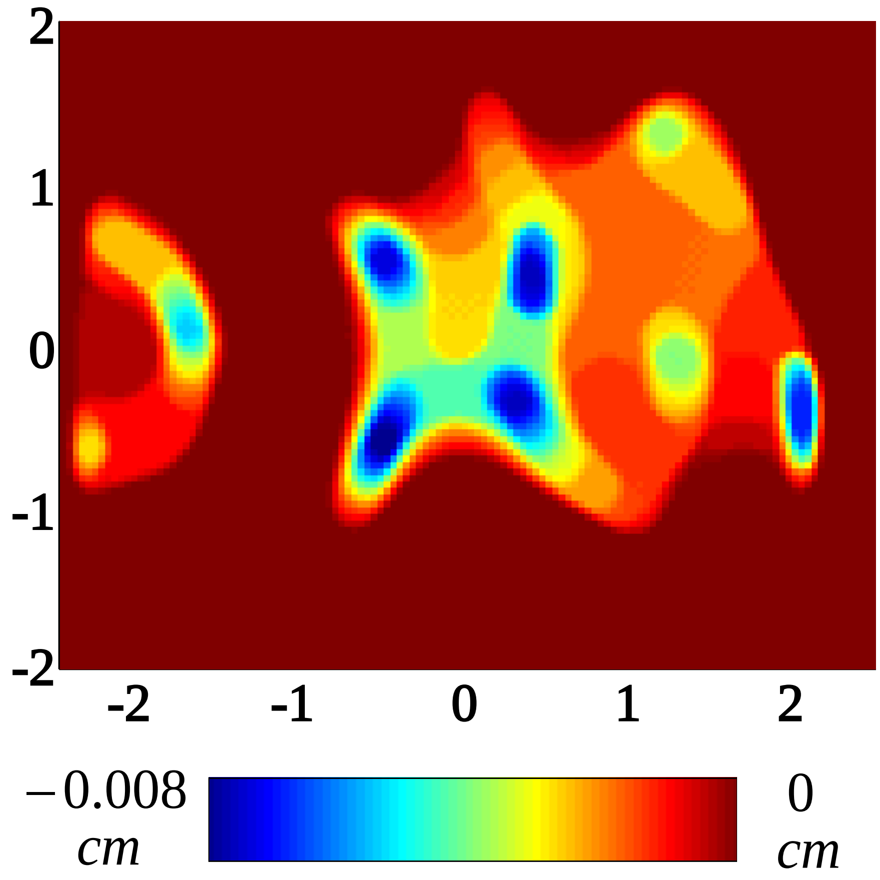
| Tissue | c $cm/ps$ | $\mu_a$ $cm^{-1}$ | $\delta\mu_a$ $cm^{-1}$ | $\mu'_s$ $cm^{-1}$ | $\delta\mu'_s$ $cm^{-1}$ | $D$ $cm$ | $\delta D$ $cm$ |
|---|---|---|---|---|---|---|---|
| **Fat** | 0.0214 | 0.050 | 0 | 10 | 0 | 0.0332 | 0 |
| **Tumor** | 0.0214 | 0.075 | 0.025 | 13 | 3 | 0.0255 | -0.0077 |
| **Fibrous** | 0.0214 | 0.060 | 0.010 | 11 | 1 | 0.0302 | -0.0030 |

# Result of reconstruction

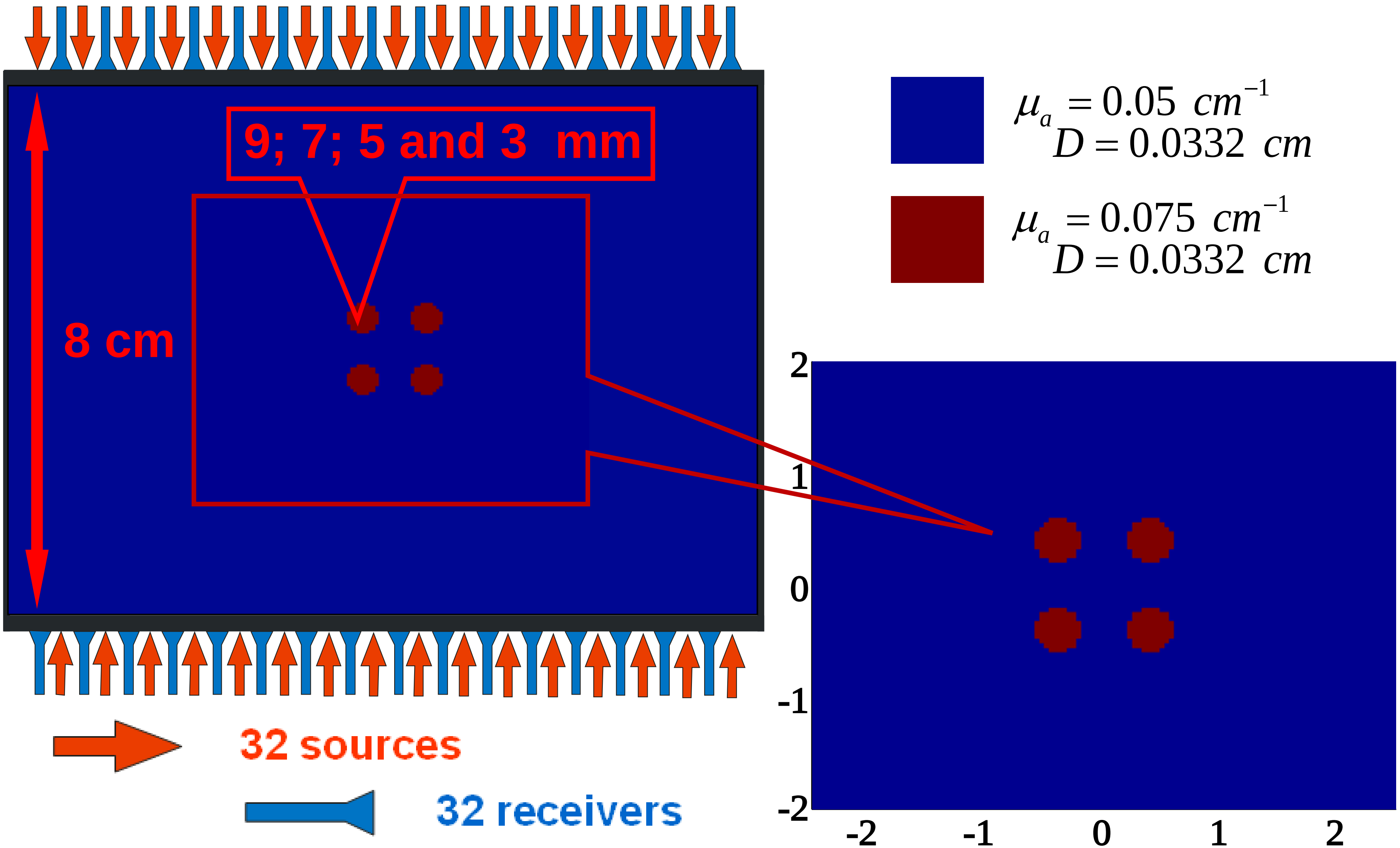
**disturbance of absorption coefficient**




**disturbance of diffusion coefficient**



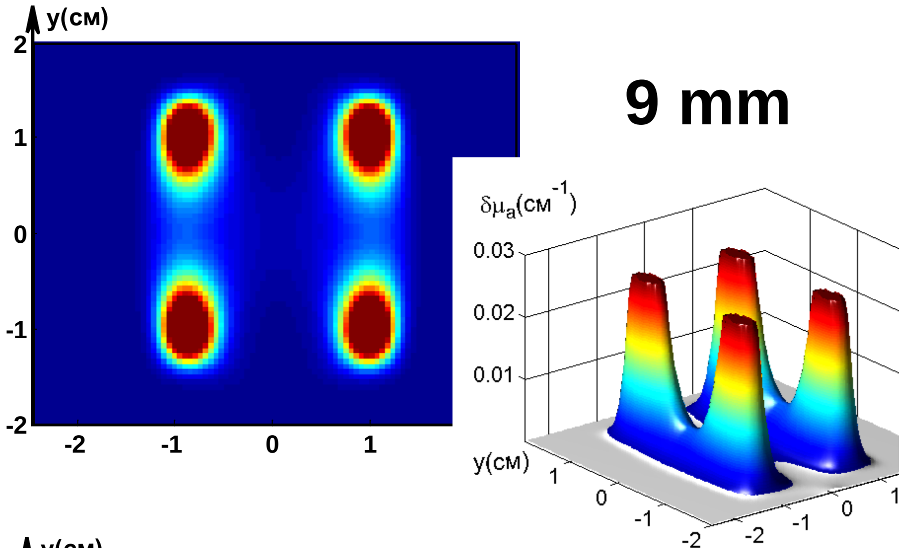
# The phantoms for evaluating resolution

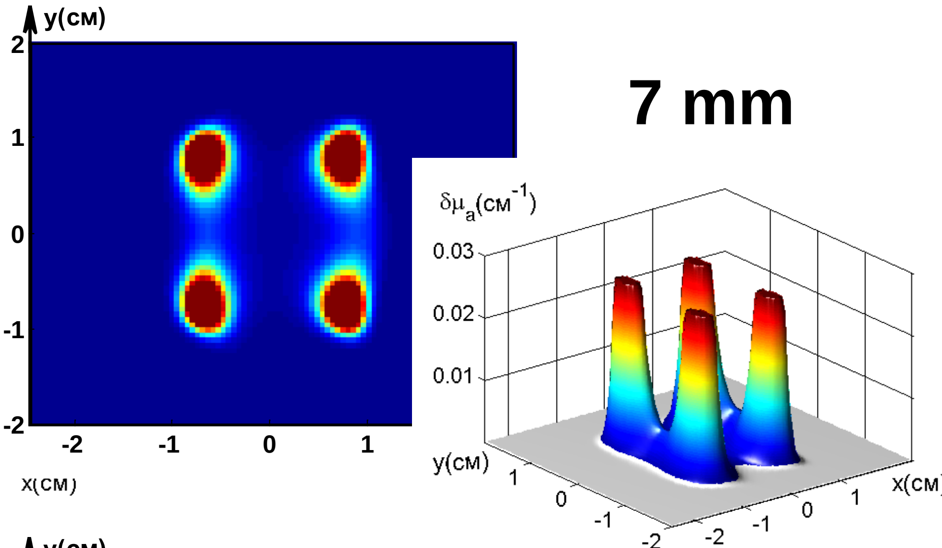


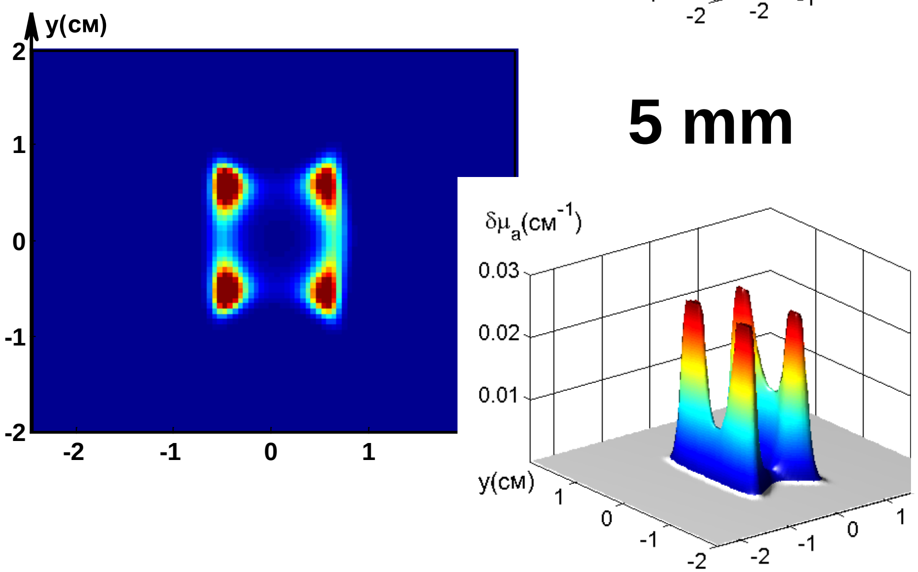

$\mu_a = 0.05\ cm^{-1}$
$D = 0.0332\ cm$

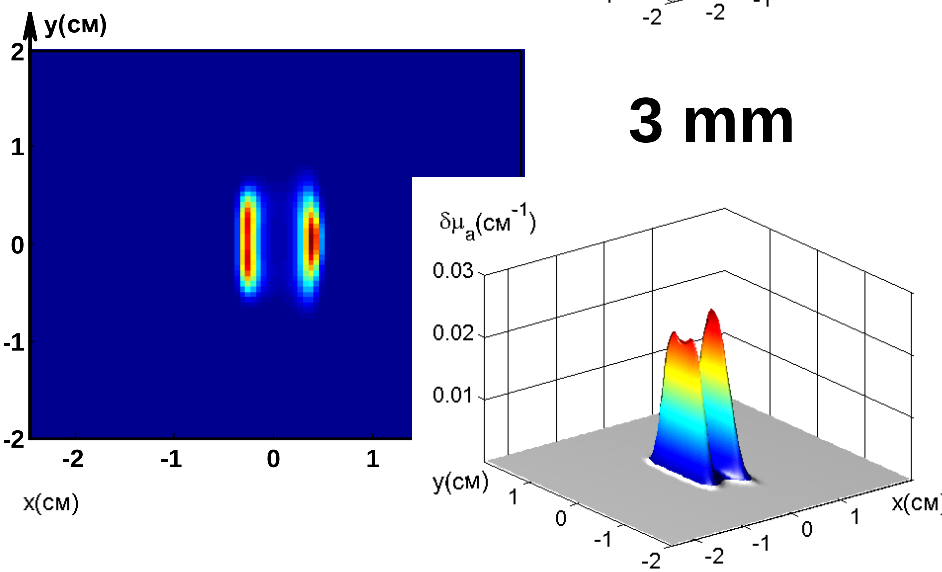
$\mu_a = 0.075\ cm^{-1}$
$D = 0.0332\ cm$

# Results of reconstruction



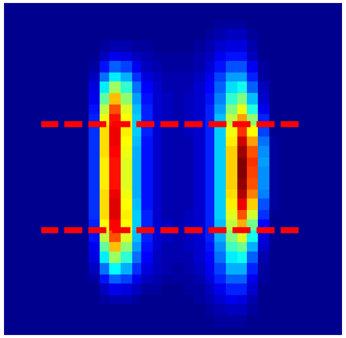



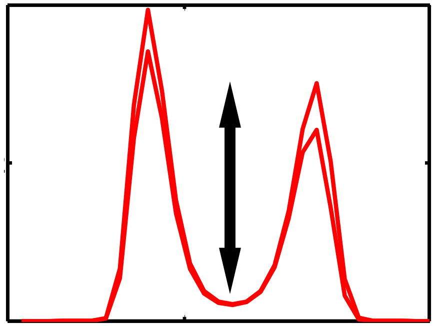



# Resolution analysis

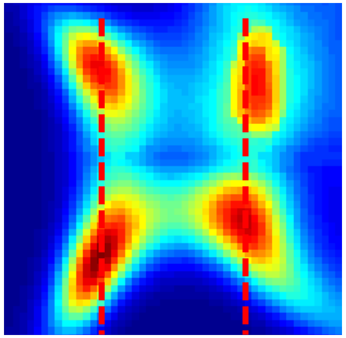
- In the centre of an 8-cm-thick object, the transverse resolution is better than 3 mm.

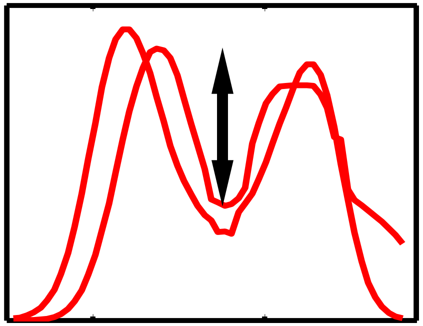




**Modulation is more than 60%**

- The longitudinal resolution is slightly worse than the transverse one. It is close to 4 mm.





**Modulation is more than 40%**

# Theoretical resolution limit

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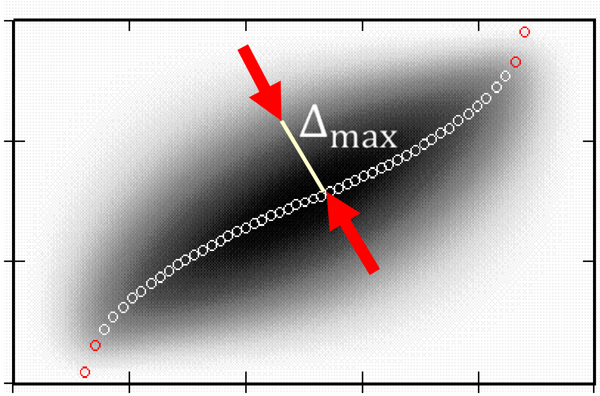



- **The photon average trajectory (PAT)**

$$\mathbf{R}(\mathbf{r}_s,t_s,\mathbf{r}_d,t)=\int_V \mathbf{r}\frac{G(\mathbf{r}-\mathbf{r}_s,t-t_s)\frac{\partial}{\partial\eta}G(\mathbf{r}_d-\mathbf{r},t_d-t)}{\frac{\partial}{\partial\eta}G(\mathbf{r}_d-\mathbf{r}_s,t_d-t_s)}d^3r$$

- **The root-mean-square deviation of photons from the PAT**

$$\Delta(\mathbf{r}_s,t_s,\mathbf{r}_d,t)=\left[\int_V |\mathbf{r}-\mathbf{R}(\mathbf{r}_s,t_s,\mathbf{r}_d,t)|^2\frac{G(\mathbf{r}-\mathbf{r}_s,t-t_s)\frac{\partial}{\partial\eta}G(\mathbf{r}_d-\mathbf{r},t_d-t)}{\frac{\partial}{\partial\eta}G(\mathbf{r}_d-\mathbf{r}_s,t_d-t_s)}d^3r\right]^{1/2}$$

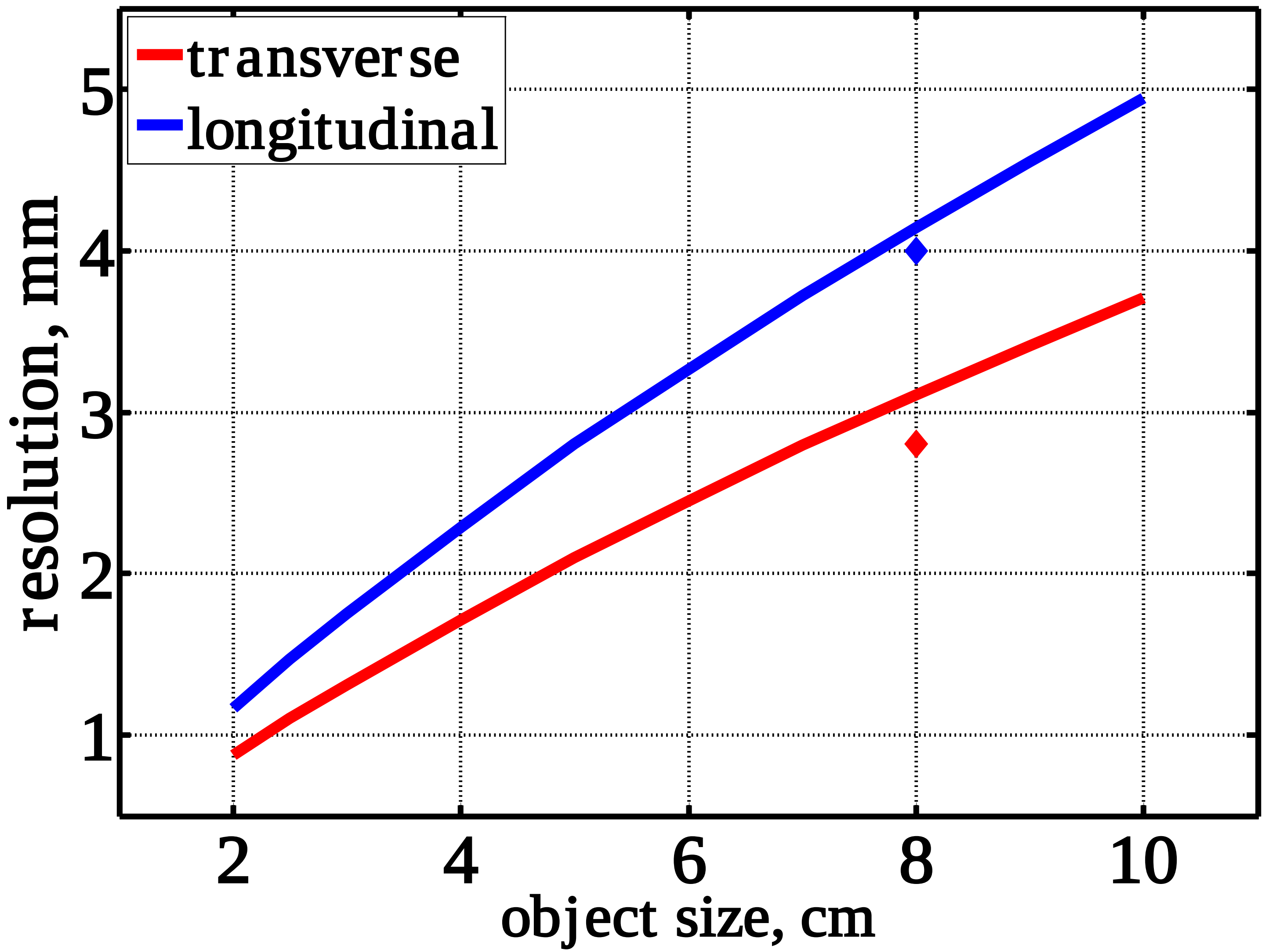
- **The resolution limit**

$$\delta_{\lim}\cong\xi\cdot\Delta_{\max}$$

V.V. Lyubimov, Opt. Spectrosc. 86: 251 (1999)

A.B. Konovalov, V.V. Vlasov, Quantum Electron. 44: 239 (2014)

# Theoretical resolution limit



# Conclusions

- For parallel-plate transmission geometry, the weight functions responsible for optical inhomogeneity reconstruction can be evaluated semi-analytically with the equivalent inverse source method.
- Our implementation of Lyubimov's perturbation model allows a record-breaking spatial resolution to be achieved (better than 3 and 4 mm inside an 8-cm-size object in transverse and longitudinal directions, respectively).

# Publications we refer to

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# The authors thank you for your time!

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